8.5 Exercise Set

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Concept Reinforcement *In each of Exercises 1–8, match the equation with a substitution from the column on the right that could be used to reduce the equation to quadratic form.*

1. (f) $4x^6 - 2x^3 + 1 = 0$ — a) $u = x^{-1/3}$
2. (d) $3x^4 + 4x^2 - 7 = 0$ — b) $u = x^{1/3}$
3. (h) $5x^8 + 2x^4 - 3 = 0$ — c) $u = x^{-2}$
4. (b) $2x^{2/3} - 5x^{1/3} + 4 = 0$ — d) $u = x^2$
5. (g) $3x^{4/3} + 4x^{2/3} - 7 = 0$ — e) $u = x^{-2/3}$
6. (a) $2x^{-2/3} + x^{-1/3} + 6 = 0$ — f) $u = x^3$
7. (e) $4x^{-4/3} - 2x^{-2/3} + 3 = 0$ — g) $u = x^{2/3}$
8. (c) $3x^{-4} + 4x^{-2} - 2 = 0$ — h) $u = x^4$

Write the substitution that could be used to make each equation quadratic in u.

9. For $3p - 4\sqrt{p} + 6 = 0$, use $u =$ $\sqrt{p}$.
10. For $x^{1/2} - x^{1/4} - 2 = 0$, use $u =$ $x^{1/4}$.
11. For $(x^2 + 3)^2 + (x^2 + 3) - 7 = 0$, use $u =$ $x^2 + 3$.
12. For $t^{-6} + 5t^{-3} - 6 = 0$, use $u =$ t^{-3}.
13. For $(1 + t)^4 + (1 + t)^2 + 4 = 0$, use $u =$ $(1 + t)^2$.
14. For $w^{1/3} - 3w^{1/6} + 8 = 0$, use $u =$ $w^{1/6}$.

Solve.

15. $x^4 - 5x^2 + 4 = 0$ $\pm1, \pm2$
16. $x^4 - 10x^2 + 9 = 0$ $\pm1, \pm3$
17. $x^4 - 9x^2 + 20 = 0$ $\pm\sqrt{5}, \pm2$
18. $x^4 - 12x^2 + 27 = 0$ $\pm\sqrt{3}, \pm3$
19. $4t^4 - 19t^2 + 12 = 0$ $\pm\frac{\sqrt{3}}{2}, \pm2$
20. $9t^4 - 14t^2 + 5 = 0$ $\pm\frac{\sqrt{5}}{3}, \pm1$
21. $w + 4\sqrt{w} - 12 = 0$ 4
22. $s + 3\sqrt{s} - 40 = 0$ 25
23. $(x^2 - 7)^2 - 3(x^2 - 7) + 2 = 0$ $\pm2\sqrt{2}, \pm3$
24. $(x^2 - 2)^2 - 12(x^2 - 2) + 20 = 0$ $\pm2\sqrt{3}, \pm2$
25. $r - 2\sqrt{r} - 6 = 0$ $8 + 2\sqrt{7}$
26. $s - 4\sqrt{s} - 1 = 0$ $9 + 4\sqrt{5}$
27. $(1 + \sqrt{x})^2 + 5(1 + \sqrt{x}) + 6 = 0$ No solution
28. $(3 + \sqrt{x})^2 + 3(3 + \sqrt{x}) - 10 = 0$ No solution
29. $x^{-2} - x^{-1} - 6 = 0$ $-\frac{1}{2}, \frac{1}{3}$
30. $2x^{-2} - x^{-1} - 1 = 0$ $-2, 1$
31. $4y^{-2} - 3y^{-1} - 1 = 0$ $-4, 1$
32. $m^{-2} + 9m^{-1} - 10 = 0$ $-\frac{1}{10}, 1$
33. $t^{2/3} + t^{1/3} - 6 = 0$ $-27, 8$
34. $w^{2/3} - 2w^{1/3} - 8 = 0$ $-8, 64$
35. $y^{1/3} - y^{1/6} - 6 = 0$ 729
36. $t^{1/2} + 3t^{1/4} + 2 = 0$ No solution
37. $t^{1/3} + 2t^{1/6} = 3$ 1
38. $m^{1/2} + 6 = 5m^{1/4}$ $16, 81$
39. $(10 - \sqrt{x})^2 - 2(10 - \sqrt{x}) - 35 = 0$ $9, 225$
40. $(5 + \sqrt{x})^2 - 12(5 + \sqrt{x}) + 33 = 0$ $4 + 2\sqrt{3}$
41. $16\left(\frac{x-1}{x-8}\right)^2 + 8\left(\frac{x-1}{x-8}\right) + 1 = 0$ $\frac{12}{5}$
42. $9\left(\frac{x+2}{x+3}\right)^2 - 6\left(\frac{x+2}{x+3}\right) + 1 = 0$ $-\frac{3}{2}$
43. $x^4 + 5x^2 - 36 = 0$ $\pm2, \pm3i$
44. $x^4 + 5x^2 + 4 = 0$ $\pm i, \pm2i$
45. $(n^2 + 6)^2 - 7(n^2 + 6) + 10 = 0$ $\pm i, \pm2i$
46. $(m^2 + 7)^2 - 6(m^2 + 7) - 16 = 0$ $\pm1, \pm3i$

Find all x-intercepts of the given function f. If none exists, state this.

47. $f(x) = 5x + 13\sqrt{x} - 6$ $\left(\frac{4}{25}, 0\right)$
48. $f(x) = 3x + 10\sqrt{x} - 8$ $\left(\frac{4}{9}, 0\right)$
49. $f(x) = (x^2 - 3x)^2 - 10(x^2 - 3x) + 24$ ⊡
50. $f(x) = (x^2 - 6x)^2 - 2(x^2 - 6x) - 35$ $(-1, 0), (1, 0), (5, 0), (7, 0)$
51. $f(x) = x^{2/5} + x^{1/5} - 6$ $(-243, 0), (32, 0)$
52. $f(x) = x^{1/2} - x^{1/4} - 6$ $(81, 0)$

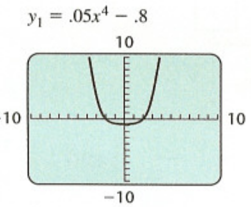
⊡ Answer to Exercise 49 is on p. IA-17.

53. $f(x) = \left(\frac{x^2+2}{x}\right)^4 + 7\left(\frac{x^2+2}{x}\right)^2 + 5$ No x-intercepts

54. $f(x) = \left(\frac{x^2+1}{x}\right)^4 + 4\left(\frac{x^2+1}{x}\right)^2 + 12$ No x-intercepts

TW 55. To solve $25x^6 - 10x^3 + 1 = 0$, Margaret lets $u = 5x^3$ and Murray lets $u = x^3$. Can they both be correct? Why or why not?

TW 56. While trying to solve $0.05x^4 - 0.8 = 0$ with a graphing calculator, Carmela gets the following screen. Can Carmela solve this equation with a graphing calculator? Why or why not?



SKILL REVIEW

To prepare for Section 8.6, review graphing functions (Sections 1.5 and 2.2).

Graph. [1.5], [2.2]

57. $f(x) = x$ ⊡

58. $g(x) = x + 2$ ⊡

59. $h(x) = x - 2$ ⊡

60. $f(x) = x^2$ ⊡

61. $g(x) = x^2 + 2$ ⊡

62. $h(x) = x^2 - 2$ ⊡

⊡ Answers to Exercises 57–62, 73, and 74 are on p. IA-17.

SYNTHESIS

TW 63. Describe a procedure that could be used to solve any equation of the form $ax^4 + bx^2 + c = 0$.

TW 64. Describe a procedure that could be used to write an equation that is quadratic in $3x^2 - 1$. Then explain how the procedure could be adjusted to write equations that are quadratic in $3x^2 - 1$ and have no real-number solution.

Solve.

65. $5x^4 - 7x^2 + 1 = 0$ $\pm\sqrt{\frac{7 \pm \sqrt{29}}{10}}$

66. $3x^4 + 5x^2 - 1 = 0$ $\pm\sqrt{\frac{-5 \pm \sqrt{37}}{6}}$

67. $(x^2 - 4x - 2)^2 - 13(x^2 - 4x - 2) + 30 = 0$ $-2, -1, 5, 6$

68. $(x^2 - 5x - 1)^2 - 18(x^2 - 5x - 1) + 65 = 0$ $-2, -1, 6, 7$

69. $\frac{x}{x-1} - 6\sqrt{\frac{x}{x-1}} - 40 = 0$ $\frac{100}{99}$

70. $\left(\sqrt{\frac{x}{x-3}}\right)^2 - 24 = 10\sqrt{\frac{x}{x-3}}$ $\frac{432}{143}$

71. $a^5(a^2 - 25) + 13a^3(25 - a^2) + 36a(a^2 - 25) = 0$ $-5, -3, -2, 0, 2, 3, 5$

72. $a^3 - 26a^{3/2} - 27 = 0$ 9

73. $x^6 - 28x^3 + 27 = 0$ ⊡

74. $x^6 + 7x^3 - 8 = 0$ ⊡

Try Exercise Answers: Section 8.5

15. $\pm 1, \pm 2$ **21.** 4 **27.** No solution **31.** $-4, 1$ **33.** $-27, 8$

49. $\left(\frac{3}{2} + \frac{\sqrt{33}}{2}, 0\right), \left(\frac{3}{2} - \frac{\sqrt{33}}{2}, 0\right), (4, 0), (-1, 0)$

Mid-Chapter Review

We have discussed four methods of solving quadratic equations:

- factoring and the principle of zero products;
- the principle of square roots;
- completing the square;
- the quadratic formula.

Any of these may also be appropriate when solving an applied problem or an equation that is reducible to quadratic form.